Dynamic Modelling of Heart Beat

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Two recently introduced methods to analyze heart beat are checked by applying them to numerically produced time series representing artificial heart beat. The phase space diagram method (G. E. Morfill, G. Schmidt) depends on the variability of the sinus rhythm and the coupling interval of the extrasystoles. The risk index method (J. Kurths et al.) seems to measure different aspects of heart beat.

1. Introduction

"Sudden cardiac death" (SCD) is one of the most frequent causes of death in industrialized western societies. Physicists and medics are searching for new methods to analyze ECG-data. The aim is to recognize patients with a high risk of SCD as early and reliably as possible. The new approaches ([1]–[6]) make use of the recently developed methods in Nonlinear Dynamics to analyze complex time series. They are based on discrete sequences $(x_i)_{i=0}^n$, the x_i denoting time intervals between two successive heart beats, more precisely the length of the i-th R-peak–R-peak–interval.

We check the new methods by applying them to numerically created data. These are obtained either by iterating a discrete map or by a non-iterative model, both producing time series $(x_i)_{i=0}^n$, which are considered as sequences of artificial heart beat intervals. The aim is to understand, which properties of such time series are responsible for which response to the analyzers. Of course, generating artificial RR-interval sequences is by no means unique. The models we introduce are chosen to be plausible and as simple as possible.

2. Phase Space Diagrams

2.1 The Method

According to Takens (see [7]) all information about a dynamical system can be extracted from every variable which contributes to the dynamics of the system's state, if the time series is long enough and the variable can be measured with sufficient precision. The phase space diagram method is based on this idea (see [4]). The data sets used by Morfill and Schmidt contain about 100 000 heart beats, the recording length was typically 24 hours. The phase space is constructed then by the time delay method. The points P_i of the phase space are defined by three successive RR-intervals as their cartesian coordinates, $P_i = (x_i, x_{i+1}, x_{i+2})$, for all $i = 1, \ldots, n-2$. The set of all P_i generated from the ECG by this method forms a cloud of points in the three-dimensional phase space. For healthy persons this point set turns out to be a cudgel along the diagonal, for the long-time-survivors there appear several cudgels that merge at their origin. Patients who later died of SCD usually show a diffuse cloud, nearly without any structure (see [1], [4]–[6]).

To quantify the observed differences between phase space diagrams of healthy persons, long time survivors with disturbances of heart rhythm and SCD-risk patients, the group of Morfill and Schmidt introduced the Scaling Index Method (SIM, see [4]). For each point they determine the number $N(r)$ of points in a sphere of radius r around it. In a certain interval $[r_1, r_2]$ the function $N(r)$ is approximated by a power law $N(r) \propto r^\alpha$. $\alpha > 0$ is called the local scaling index. Plotting $f(\alpha)$, the frequency of α, versus α in a histogram, they find two maxima in this plot. As the distance $\Delta\alpha$ between these maxima becomes the larger, the more diffuse the cloud of points in the phase space, $\Delta\alpha$ can be used as a quantitative risk parameter (see [4]).

2.2 Sinus Rhythm

The most natural guess for an artificial heart beat sequence is that all intervals x_i are equal except of being polluted by noise of physiological origin, for

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example of the vegetative nervous system. This idea is reflected in the map $x_{i+1}=x_i+\sigma\xi_i$. Here ξ_i is normalized gaussian white noise, i.e., $\langle\xi_i\rangle=0$, $\langle\xi_i\xi_j\rangle=\delta_{ij}$, and σ its strength. In order to prevent the signal sequence x_i from diffusional broadening we add a restoring term $-c(x_i-1)$:

$$x_{i+1}=x_i-c(x_i-1)+\sigma\xi_i, \quad x_i\in[0,2], \quad i=0,1,2,\ldots,n, \quad c\ll\sigma. \tag{1}$$

c should be small compared with σ, otherwise the restoring term would disturb the desired dynamics.

This model generates a diagonal cudgel in the phase space diagram (see Fig. 1) being similar to that which Morfill and Schmidt found for healthy persons (see [1], [4]–[6]).

2.3 Extrasystoles With Fixed Coupling

Extrasystoles are extra beats of the heart. Unlike for the sinus rhythm their electrical origin is not the sinus node. We use the following algorithm to include extrasystoles with compensatory pause and fixed coupling interval:

- If the last heart beat was an extrasystole, then a compensatory pause will follow.
- If the last heart beat was *not* an extrasystole, then the next one will again be a sinus beat with probability $p=0.75$ or an extrasystole with probability $1-p=0.25$.

This algorithm generates a sequence like "$\ldots x\,x\,y\,2x-y\,x\,y\,2x-y\,x\,x\ldots$". Here x denotes the distance between two successive sinus beats, y the time lag between an extrasystole and the preceding beat, denoted as coupling interval. So $2x-y$ is the length of the compensatory pause to follow. In this section we consider $y=y_0$ constant (fixed coupling interval) while the regular beats are again modelled by the $(x_i)_{i=0}^{n}$ from (1).

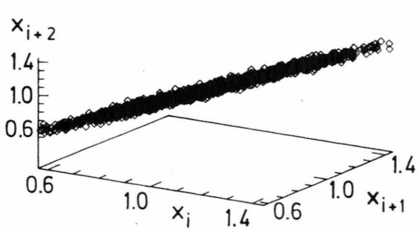

Fig. 1. Phase space diagram of model (1) with parameters $\sigma=0.02$, $c=0.003$, $x_0=1$, and $n=2000$. (Even if we take $n=100\,000$ the cudgel is well within [0, 2]. It does not depend on x_0.)

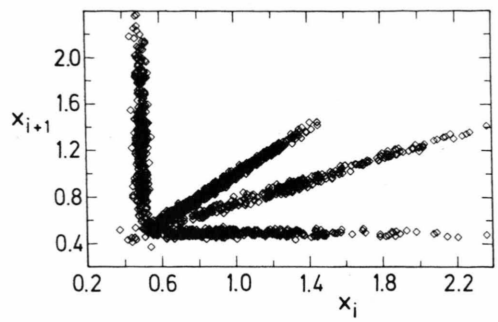

Fig. 2. Projection of the full phase space diagram in Fig. 3 on the x_{i+1}–x_i-coordinate plane. Noisy regular beat with fixed extrasystoles and compensatory pause, $n=2000$, $\sigma=0.02$, $c=0.003$, $y_0=0.5$, and $p=0.75$.

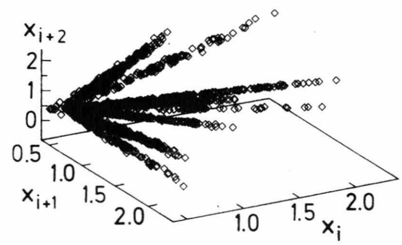

Fig. 3. Phase space diagram for variable sinus rhythm and extrasystoles with compensatory pause and fixed coupling. Parameters as in Figure 2.

One obtains a phase space cloud with four cudgels in the projection on the x_{i+1}–x_i-coordinate plane (see Figure 2). One cudgel is not symmetric with respect to the angle bisector, because the compensatory pause breaks the symmetry of the sequence "$\ldots x\,y_0\,2x-y_0\,x\ldots$". In the three-dimensional phase space diagrams there are eight different cudgels (see Fig. 3) corresponding to the following eight patterns:

$$\begin{array}{ccccccc} x & x & x\,, & \quad & 2x-y_0 & x & x\,,\\ x & x & y_0\,, & & 2x-y_0 & x & y_0\,,\\ x & y_0 & 2x-y_0\,, & & y_0 & 2x-y_0 & y_0\,,\\ y_0 & 2x-y_0 & x\,, & & 2x-y_0 & y_0 & 2x-y_0\,. \end{array}$$

This phase space structure is typical for the ECG generated point set of long-time-survivors, as reported in [1], [4]–[6].

2.4 Extrasystoles With Variable Coupling

When choosing, on the other hand, a constant sinus rhythm x_0 but a variable coupling interval y, one obtains a phase space diagram which agrees very well

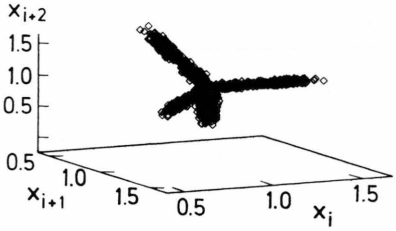

Fig. 4. Phase space diagram for constant sinus rhythm $x_0(=1)$, but variable coupling interval y. Parameters for model (1): $\sigma=0.02$, $c=0.003$, $y_0=0.5$. In this special case we did not use the $(p, 1-p)$-algorithm but the one constantly alternating three extrasystoles and one sinus beat.

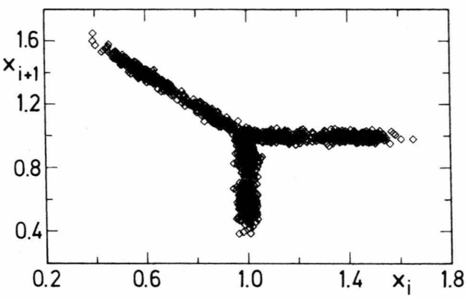

Fig. 5. Projection of the full phase space diagram in Fig. 4 on the $x_{i+1}-x_i$-coordinate plane.

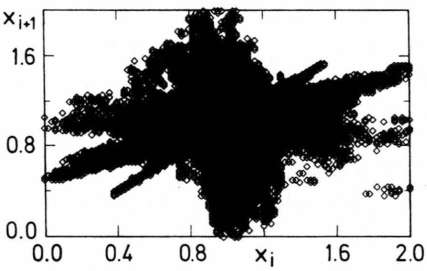

Fig. 6. Phase space diagram for variable sinus rhythm and variable coupling of the extrasystoles. Parameters: $x_0=1$, $y_0=0.5$, $\sigma=0.01$, $c=0.003$, $n=100000$. Obviously there is a small cudgel of pure sinus beats in the middle of the cloud as observed by Morfill and Schmidt, too.

with that of a sick person shown in [6], p. 161. It contains only four cudgels (see Figure 4). We now describe the successive coupling intervals y by a model sequence generated by (1) and keep $x=x_0=$const. In this special case three sinus beats (or more) and one extrasystole are alternating, so the generated sequence is "$\ldots x_0\,x_0\,y\,2x_0-y\,x_0\,x_0\,y\,2x_0-y\ldots$". Otherwise (if there can appear less than three sinus beats between two extrasystoles as in Sect. 2.3), there would appear more than four cudgels in phase space, namely in addition the cudgels $2x_0-y\,x_0\,y$, $y\,2x_0-y\,y$ and $2x_0-y\;y\;2x_0-y$. The four cudgels correspond to the triples $x_0\,x_0\,y$, $x_0\,y\,2x_0-y$, $y\,2x_0-y\,x_0$ and $2x_0-y\,x_0\,x_0$. In the $x_{i+1}-x_i$-projection only three cudgels appear (see Fig. 5), corresponding to $x_0\,y$, $y\,2x_0-y$ and $2x_0-y\,x_0$, because $x_0\,x_0$ is just one point.

How would the phase space clouds $\{P_i\}$ look like if both, x and y, are variable? We generated x as well as y by the simple noise polluted model (1). For the extrasystole occurrence we again used the $(p, 1-p)$-algorithm of Sect. 2.3 with $p=0.75$. The point set generated by this twofold variable noisy sequence of x's and y's produces a three-dimensional patch (see Figure 6). It resembles the three dimensional diffuse patch in phase space which was observed by Morfill and Schmidt in the diagrams of patients who later died from SCD ([1], [4]–[6]).

2.5 Conclusion

We conclude that the phase space structures observed in [1], [4]–[6] do not seem to present more than noise polluted regular heart beat, with a small restoring force, tying the length of the RR-intervals to a fixed value, and with occasional extrasystoles, whose regular or irregular occurance is reflected in the complexity of the structures in the phase space. More specific information about SCD then should perhaps be looked for in finer details of the beat sequences $(x_i)_{i=0}^{n}$. Interesting for further investigation is the question if our models are also in quantitative accordance with the Scaling Index Method.

3. Risk Index Method

3.1 The Method

In contradistinction to the phase space diagrams, this method is based on ECG-records with a duration of 30 minutes while the patient is at rest. The examined persons are classified in healthy, low risk, and high risk persons, depending on the value of a risk index. The risk index R is the number of failures of six different tests. A person with risk indices 0 or 1 is called "healthy", one with risk indices 2, 3 or 4 is said to have a low risk, index 5 is between low and high risk, while risk index 6 is defined as "high risk" (see [2]). As a standard to calibrate the method, Kurths et al.

did choose a conventional method for evaluating SCD risk, the classification of ventricular extrasystoles according to Lown (see e.g. [8]). This classification is based on the number and morphology of extrasystoles in an ECG, on the presence or absence of a regular 1:1 pattern of sinus beats and extrasystoles, of repeated, and of "early", untimely extrasystoles. Lown's classification comprises six classes, from 0 (no arrhythmia) to 5 (untimely extrasystoles).

The six risk criteria included in R according to [2] are:

i) The quotient $Q=\frac{q}{\mu}$ of the standard deviation q and of the mean value μ of the beat sequences $(x_i)_{i=0}^n$ shall be in the interval (0.044, 0.082) for healthy persons.
ii) The deviation A of the probability density function f corresponding to the sequence $(x_i)_{i=0}^n$ from the gaussian form shall be smaller than 0.145 for healthy persons. The deviation is defined by

$$A = \int_{\mu-5q}^{\mu+5q} |f_{\text{real}}(x) - f_{\text{gauss}}(x)|\,dx\,.$$

Here again μ denotes the mean value of the sequence $(x_i)_{i=0}^n$ and q the standard deviation of the beat intervals $(x_i)_{i=0}^n$.

The data-sets $(x_i)_{i=0}^n$ are transformed into other sequences $(s_i)_{i=1}^{n-1}$ containing only a few symbols s_i, which reflect the most important properties of the heart beat sequence ("symbolic dynamics"). The s_i are defined as follows:

$$s_i = \begin{cases} 0 : x_i - x_{i+1} > 0 & \text{and} \quad |x_i - x_{i-1}| \le q^2, \\ 1 : x_i - x_{i+1} > 0 & \text{and} \quad |x_i - x_{i-1}| > q^2, \\ 2 : x_i - x_{i+1} \le 0 & \text{and} \quad |x_i - x_{i-1}| \le q^2, \\ 3 : x_i - x_{i+1} \le 0 & \text{and} \quad |x_i - x_{i-1}| > q^2. \end{cases}$$

This definition of $s_i = 0, 1, 2$ or 3 attributes the following properties: If there are large jumps in the sequence of RR-intervals (large means large in comparison with the variance q^2 of the whole sequence) an odd number is used as the symbol, otherwise an even one. Any subsequence of m successive s-symbols is called a "word". Here in particular words consisting of $m=3$ symbols are considered. This choice is made to compromise long and short words appropriately; long words are better representatives of the long term dynamics, short words are favorable for estimating the frequency of the symbols. We wonder, why the authors of [2] compare $|x_i - x_{i+1}|$ with q^2. As q has the same dimension as $|x_i - x_{i-1}|$, it seems to be more sensible to compare these two quantities. The following criteria included in R are based on the symbolic dynamics $(s_i)_{i=1}^{n-1}$:

iii) At least seven words (of length $m=3$) containing only even symbols must be among the ten words with highest probability of occurrence in $(s_i)_{i=1}^{n-1}$.
iv) There should be at least 25 words with a probability smaller than 0.001 ("forbidden words").
v) The Shannon-information calculated for words of length three, $S_3 = -\frac{1}{6}\sum_{s^3} p(s^3)\log_2 p(s^3)$, should be in the interval $S_3 \in (0.510, 0.602)$. Here $p(s^3)$ denotes the probability of the word s^3.

The sixth criterium for health is chosen to be

vi) Klimontovich's renormalized entropy K is in $(-0.7, 0)$. For the definition of K see [9]; we omit it, since we did not include K in our analysis, cf. end of Section 3.2.

3.2 Sinus Rhythm

We checked these risk criteria for the artificial heart beat sequences $(x_i)_{i=0}^n$ defined in Sect. 2 and found that model (1) always gives a risk index larger than or equal to two, for all reasonable choices of the parameters σ and c. So model (1) does not describe the time series of healthy persons according to the risk index R, at least for these short sequences ($n=2000$) (for more details see [10]), because "healthy" sequences should have a risk index 0 or 1. We remark that it is not the failure of always the same criteria which contributes to the risk index.

For this check we used the definition of the s_i as it was made in [2]. If we substitute q^2 by q in this definition, we can get a risk index of 0 or 1 corresponding to a healthy person, e.g. with the parameters $\sigma=0.02$ and $c=0.05$.

In the following we consider another model. Its aim is to achieve a healthy risk index even for the definition of symbolic dynamics made by Kurths et al.

$$x_i = \bar{x}_i + \sigma\,\bar{x}_i\,\xi_i\,, \qquad i=0,1,\ldots,n\,, \tag{2}$$

with

$$\bar{x}_i = 2(b+a)(1-\tfrac{i}{n})^3 - 3(b+a)(1-\tfrac{i}{n})^2 + a(1-\tfrac{i}{n}) + 1 \in [1-b, 1] \quad \text{and} \quad -3b \le a \le -b \le 0\,.$$

It consists of a deterministic function $\bar{x}_i$ describing the time dependent mean value of the heart beat interval length and a noise term $\sigma\,\bar{x}_i\,\xi_i$. Again ξ_i is normalized gaussian white noise, $\sigma\,\bar{x}_i$ its strength. b determines the shortest possible length $1-b$ of RR-intervals. $\bar{x}_i$ describes a heart beating rather fast at the beginning and slowing down during the record.

The function $\bar{x}_i$ versus i fulfils the conditions

1. $\bar{x}_0 = 1 - b$ and $\bar{x}_n = 1$
2. $\bar{x}_i$ is monotonously increasing
3. $\bar{x}_i$ has a point of inflection at $i = \frac{n}{2}$, i.e., $\bar{x}''_{i=\frac{n}{2}} = 0$, and $\bar{x}'''_{i=\frac{n}{2}} \geq 0$. (Consider i as a continuous variable here to give a meaning to the derivatives.)

The parameter a determines the slope at the inflection point. Model (2) seems to contain some important properties of a healthy heart. While in model (1) both parameters σ and c influence all six criteria simultaneously, here we can adjust the parameters σ, a, and b more independently and adapt to one criterion after the other. This can be done as follows: The choice of b fixes the variance of $(x_i)_{i=0}^{n}$, if the strength σ of the noise is small, i.e., $\sigma \ll 1$. So criterion i) can be met. Next, we choose the parameter a to achieve criterion ii). The optimal a depends on b, it seems to be about $-2b$. The smaller σ, the more often are even symbols. Thus, σ mainly influences the criteria derived from symbolic dynamics, iii) to v). For the parameters $b = 0.17$, $a = 0.33$, $n = 2000$ and $\sigma = 0.0005$ five of the six risk criteria are achieved (see [10]). We did not calculate the sixth criterion (the Klimontovich entropy K), because it does not change the interpretation of the risk indices we obtain. (Remember, that $R = 5$ is as well in the class of low as of high risk.) In both cases, criterion vi) being fulfilled or not, extrasystoles turn out not to be responsible for a high risk classification, as will be detailed now.

3.3 *Influence of Extrasystoles*

Having defined model (2) for the sinus rhythm, we can check the influence of embedded extrasystoles with constant coupling interval on the risk index R. The effects of extrasystoles with fixed coupling and compensatory pause are shown in the upper half of Table 1. The lower value of the risk index in the table is obtained if the sixth criterion would be satisfied, otherwise we get the higher value.

To model the extrasystoles with variable coupling interval we used two rhythms like

$$x_{i+1} = x_i - c_x(x_i - 1) + \sigma_x \xi_i \quad \text{for the sinus rhythm,}$$

$$y_{i+1} = y_i - c_y(y_i - 1) + \sigma_y \eta_i \quad \text{for the extrasystoles}$$

(see Section 2.4). Because the part of this model describing sinus rhythm does not fulfil all criteria for health simultaneously, we took different parameter sets for the check of (i), (ii) and for (iii)–(v), namely, $\sigma_x = 0.02$ and $c_x = 0.05$ or $\sigma_x = 0.01$ and $c_x = 0.0002$. With these two sets of parameters the first five criteria can be met for model (1) (see [10]). Additionally, we choose $\sigma_y = \sigma_x$ and $c_y = c_x$ because this choice produced sensible results for the phase space diagram method. The results for extrasystoles with compensatory pause and variable coupling are shown in the lower half of Table 1. The most important observation is that in no case all criteria are violated (highest risk index 6). Thus, neither extrasystoles with fixed nor with variable coupling can be responsible for the classification as a high risk patient. There must be an additional property in the heart beat sequence of highest risk patients, which are not yet included in our simple artificial sequences according to models (1) or (2).

Table 1. A) The dependence of the risk index on the number of extrasystoles with compensatory pause and fixed coupling interval. The chosen parameters for (2) are: $\sigma = 0.0005$, $b = 0.17$, $a = -0.33$, $n = 2000$. B) Here we took model (1) with $\sigma = 0.02$, $c = 0.05$ for modelling the first two criteria and $\sigma = 0.01$, $c = 0.0002$ to satisfy the criteria of symbolic dynamics. $n = 2000$, $x_0 = 1$ always. The coupling interval is taken here as variable.

A) number of extrasystoles	0	1	6	12	18	32	93	183	327	667
risk index	0–1	1–2	2–3	2–3	1–2	2–3	3–4	3–4	4–5	3–4
B) number of extrasystoles	0	1	12	20	40	184	669			
risk index	0–1	0–1	1–2	0–1	3–4	3–4	3–4			

4. Summary and Discussion

The phase space reconstruction method depends on two features of the sequences of RR-intervals:

- fixed or variable coupling interval y,
- variability of RR-intervals x of sinus rhythm.

It is mentioned in [11] that extrasystoles with fixed coupling only seldomly trigger ventricular disorders of hearth rhythm (e.g. fibrillation). This confirms the suggestion by Morfill and Schmidt, who assign low risk to diagrams with extrasystoles of fixed coupling and high risk to those of variable coupling interval.

The risk index method obviously is not sensitive to the same parameters as the phase space method. A particular shortcoming seems to be that it does not classify patients with extrasystoles with variable coupling as being high risk patients. Kurths et al. [2] noticed a good agreement of their results with the Lown classification. It is somewhat surprising that they did not observe their patients over longer periods to find out how many of them died and how many survived. So the good agreement with the Lown classification not necessarily means that this new method is superior to the conventional one.

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